An Integrated Vendor-Buyer Model with Exponentially Increasing Demand in Fuzzy Environment

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ABSTRACT

In today's marketing scenario, the coordination between vendor and buyer is getting great importance. A large numbers of supply chain models have been developed on exact parameter values. However, in reality, these parameter values may not be reliable due to vagueness. In this article, we develop an integrated vendor-buyer model with exponentially increasing demand at the buyer. Our purpose is to design a coordination mechanism between the vendor and the buyer with incomplete information about the cost parameters. We assume the set up cost, ordering cost, transportation cost, vendor's and buyer's holding costs in the fuzzy sense. We use centroid method for defuzzification of the fuzzy average total cost of the integrated system. The optimal results of the proposed fuzzy model are obtained and sensitivity of key model-parameters is examined through a numerical example.

Keywords: Inventory, supply chain, vendor-buyer model, exponential demand, fuzzy.

INTRODUCTION:

In recent years, integrated vendor-buyer model has received a great attention from both academia and industry. Researchers have shown their keen interest on supply chain management realizing its potential to improve performance of business firms at a reduced cost and delivery time. Moreover, an efficient management of inventory across the entire supply chain through better coordination and cooperation can reduce the system cost quite significantly. This leads the researchers to develop the strategic coordination between vendor and buyer in today's supply chain management.

Goyal (1976) is one of the first to develop a simple supply chain model with single vendor and single buyer. After that, several researchers have contributed to the development of vendor-buyer coordination model. Ben-Daya and Hariga (2004) considered stochastic demand and variable lead time to develop a single vendor and single buyer model and showed that coordination is effective from the vendor's as well as the buyer's perspective. Considering stochastic demand and quantity discount policy, Li and Liu (2006) proposed a model to achieve coordination between the vendor and the buyer. Qin et al. (2007) considered volume discount and franchise fees and price sensitive demand in their single vendor single buyer model. Sajadieh et al. (2009) developed a vendor-buyer model where the production batch is delivered to the buyer in n equal shipments and the lead time is stochastic. Yang (2010) assumed variable lead time to analyze present value in supply chain model. Moharana et al. (2012) in their supply chain model investigated the stability of coordination, collaboration and integration between the buyer and the vendor. Shah et al. (2013) assumed the demand as

price-sensitive and stock-dependent and also the order-linked trade-credit in a single-supplier single-buyer inventory model to derive the optimal pricing, shipments and ordering policies. Dey and Giri (2014) considered investment in reducing the defect rate in an imperfect production system of the vendor. Giri and Bardhan (2015) studied a vendor-buyer JELS model with stock-dependent demand and consigned inventory when the buyer has capacity constraint. Zhang et al. (2016) developed a vendor-buyer model considering discount policy. Giri and Sharma (2017) employed a generalized shipment policy, error in inspection and ordering cost reduction for a vendor-buyer imperfect system. Li et al. (2017) proposed a model to investigate the environmental impacts.

Researchers considered crisp values for the cost parameters in all these works. They assumed that the cost parameter values are precise and fixed. But, in reality and in different circumferences, these costs may vary with time. To capture these uncertainties some probabilistic models have been developed. Different statistical methods have been used to derive the parameter(s) of probability distribution from the past data. However, statistical methods may face problems due to the unavailability or unreliability of the past data. Thus researchers may face the lack of cost information. To cope up with this problem, one may use fuzzy set theory rather than probability theory and represent uncertainty by possibility distribution.

Handling the vague situation is possible due to Zadeh (1965). In recent years, fuzzy set theory is being widely used in inventory and supply chain management. Yao and Lee (1996) in their back order fuzzy inventory model considered fuzzy order quantity as triangular and trapezoidal fuzzy numbers and shortage cost as a crisp parameter. Yao and Chiang (2003) proposed an inventory model without backorder where they fuzzified the total demand and cost of storing by triangular fuzzy numbers and defuzzified the total cost by centroid and signed distance methods. Chiang et al. (2005) assumed storing cost, backorder cost, ordering cost, total demand, order quantity and shortage quantity as triangular fuzzy numbers and defuzzified by signed distance method. Lin (2008) fuzzified the expected demand, shortage and backorder rate defuzzified the cost function obtained in the fuzzy sense using signed distance method. Sadi-Nezhad et al. (2011) proposed a continuous review inventory model in which they fuzzified the setup cost, holding cost and shortage cost and defuzzified by signed distance and possibilistic mean value methods. Later, Lee and Lin (2011) and Sarkar and Chakrabarti (2012) contributed by developing some fuzzy inventory models. Dey et al. (2016) developed a fuzzy random continuous review inventory model with a mixture of backorders and lost sales under imprecise chance constraint.

During the last few years, researchers have been developing vendor-buyer supply chain models under fuzzy environment. Mahata et al. (2005) considered order quantity as a fuzzy variable to develop a joint economic lot size model for both buyer and vendor. Gunasekaran et al. (2006) proposed a supply chain model to optimize the order quantity with fuzzy approach. Sucky (2006) developed a two-staged supply chain coordination model under asymmetric information where he assumed that the buyers have two sets of predefined(deterministic) cost structures and the vendor does not know in which particular cluster does a particular buyer belong. Sinha and Sarmah (2008) proposed a vendor-buyer supply chain model with quantity discount policy considering ordering cost, inventory holding cost of the buyer and demand as fuzzy variables. Later, Ganga and Carpinetti (2011), Costantino et al. (2012) put some valuable contributions in their supply chain models under fuzzy environment. Applying fuzzy order quantity and fuzzy shortage quantity Mahata (2015) proposed a single-vendor single-buyer model. Priyam and Uthayakumar (2016) considered variable lead time and service level constraint in their fuzzy supply chain model. Jauhari and Saga (2017) proposed a vendor-buyer model with set up cost reduction and service level constraint.

In this paper, we develop an integrated supply chain system consisting of a single-buyer and a single-vendor in fuzzy environment. We assume that the vendor's production rate is dependent on the buyer's demand rate which is an exponential function of time. The set up cost, ordering cost, transportation cost, holding costs for the vendor and the buyer are considered as fuzzy variables. Centroid method is used for defuzzification of the fuzzy average total cost of the integrated system. The buyer places an order quantity and the vendor delivers to the buyer in n shipments where the shipment size is increasing so that there no shortage occurs in the buyer's inventory. The objective of the study is to determine the optimal number of shipments from the vendor to the buyer in a cycle and the optimal size in each shipment so that the average total cost is minimized. The rest of the paper is organized as follows: In the next section, some assumptions and notations are given. In the subsequent section, the mathematical formulation of the model is given. The next section provides some preliminaries on fuzzy set theory and defuzzification method. The subsequent section deals with the implementation of fuzzy concept in the proposed vendor-buyer system. Then numerical example is taken to illustrate the developed models and perform sensitivity analysis with respect to some model parameters. In the last section, the paper is concluded with some remarks.

ASSUMPTION AND NOTATIONS:

The following assumptions are made for developing the proposed model:
(i)   The supply chain consists of a single buyer and a single vendor, and it considers a single type of product.
(ii) The demand rate at the buyer is exponentially increasing in time.
(iii) The vendor's production process is perfect and its production rate is greater than the buyer's demand rate.
(iv) The vendor delivers to the buyer in n unequal shipments in each cycle. Shipment to buyer is made as soon as the vendor completes production of that much amount which is equal to the size of the next shipment.
(v)   The buyer's replenishment rate is infinite and shortages are not allowed in buyer's inventory.
(vi) Starting time of the vendor's production for a particular shipment and ending time of the buyer's inventory after receiving that particular shipment from the vendor is termed, in this paper, as replenish time interval.

The following notations are used to construct the proposed model:
$d(t)$: Demand rate in units per unit time.
$p(t)$: Production rate in units per unit time; we take $p(t) = kd(t), k > 1$, a constant.
n : Number of shipments from the vendor to the buyer in a cycle.
t_r : Time delay for the vendor to start production for each shipment.
$I_i(t)$: Inventory level in the ith interval.
$I_{i_b}(I_{i_v})$: Inventory held by the buyer (vendor) throughout the ith interval.
$I_b(I_v)$: Inventory held by the buyer throughout the entire cycle.
q_i : Size of the ith shipment from the vendor to the buyer.
Q : Total ordered quantity from the buyer to the vendor.
$C_0(\tilde{C}_0)$: Set up cost of the vendor in crisp (fuzzy).
$C_1(\tilde{C}_1)$: Ordering cost of the buyer for each order in crisp (fuzzy).
$C_2(\tilde{C}_2)$: Transportation cost for the buyer for each shipment in crisp (fuzzy).
$h_b(\tilde{h}_b)$: Holding cost per unit per time at the buyer in crisp (fuzzy).
$h_v(\tilde{h}_v)$: Holding cost per unit per time at the vendor in crisp (fuzzy).
$ATC_b(\widetilde{ATC_b})$: Average total cost at the buyer in crisp (fuzzy).
$ATC_v(\widetilde{ATC_v})$: Average total cost at the vendor in crisp (fuzzy).
$ATC(\widetilde{ATC})$: Average total cost of the integrated supply chain in crisp (fuzzy).
τ : Time interval between successive replenishments at the buyer.

MATHEMATICAL FORMULATION:

The vendor-buyer relationship according our assumption can be described as follows: The vendor obtains an order of size Q from the buyer. The vendor starts his production with a finite and uniform production rate which is greater than the increasing demand rate of the buyer. The buyer receives the order quantity from the vendor in n shipments.

Buyer's perspective:
The market demand rate at the buyer is given by
$$d(t) = ae^{bt}, a > 0, b > 0, t \geq 0 \qquad (1)$$
Since the demand at the buyer is exponentially increasing in time, we assume that the delivery quantities of the vendors in different shipments are also increasing in size so that no shortage occurs in the buyer's inventory. The inventory profile for the vendor and buyer is depicted in Figure 1.

Figure 1: Inventory levels of vendor and buyer in the i-th interval
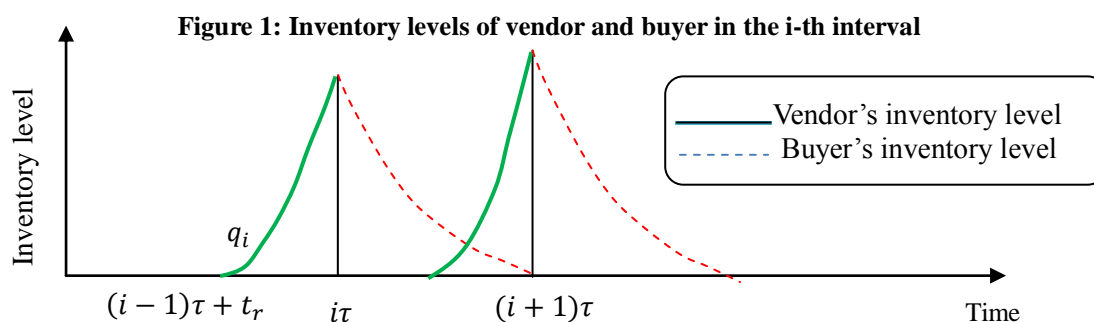
In the replenishment interval $[i\tau, (i + 1)\tau]$, the inventory level of the buyer at any time t can be expressed by the following differential equation:

$$\frac{d[I_i(t)]}{dt} = -ae^{bt}, i\tau \le t \le (i + 1)\tau; i = 1, 2, 3, \ldots, n \qquad (2)$$

with the conditions

$$I_i(i\tau) = q_{i_b} \quad (3)$$
$$I_i((i + 1)\tau) = 0 \quad (4)$$

Solving (2) with help of (4), we get

$$I_i(t) = -\frac{a}{b}e^{bt} + \frac{a}{b}e^{b(i+1)\tau} ; i = 1, 2, 3, \ldots, n \qquad (5)$$

Now, from (3) and (5), we get

$$q_{i_b} = \frac{a}{b}\left(e^{b\tau} - 1\right)e^{bi\tau} ; i = 1, 2, 3, \ldots, n \qquad (6)$$

Therefore, the total ordered quantity Q demanded by the buyer is given by

$$Q = \frac{a}{b}\left[e^{b(n+1)\tau} - e^{b\tau}\right] \qquad (7)$$

The total inventory carried out by the buyer through the ith replenishment interval is given by

$$I_{i_b} = \int_{i\tau}^{(i+1)\tau} I_i(t)dt = Ae^{bi\tau} ; i = 1, 2, 3, \ldots, n$$

where $A_1 = -\frac{a}{b^2}[e^{b\tau} - 1] + \frac{a\tau}{b}e^{b\tau}$.

Therefore, the total inventory carried out by the buyer (due to n shipments from the vendor) through the entire cycle is given by

$$I_b = \sum_{i=1}^{n} I_{i_b} = A_1\left[\frac{e^{b(n+1)\tau} - e^{b\tau}}{e^{b\tau} - 1}\right] = A_1R \qquad (8)$$

where $R = \frac{e^{b(n+1)\tau} - e^{b\tau}}{e^{b\tau} - 1}$.

Therefore, the total cost per unit time for the buyer is given by

$$ATC_b(\tau) = \frac{C_1}{n\tau} + \frac{C_2}{\tau} + \frac{h_b}{n\tau}A_1R \qquad (9)$$

Vendor's Perspective:

The variation of vendor's inventory level is depicted in Figure 1. In the replenishment interval $[(i - 1)\tau + t_r, i\tau]$, the inventory level of the vendor at any time t can be expressed by the following differential equation:

$$\frac{d[I_i(t)]}{dt} = kae^{bt}, (i - 1)\tau + t_r \le t \le i\tau; i = 1, 2, 3, \ldots, n \qquad (10)$$

with the conditions

$$I_i((i - 1)\tau + t_r) = 0 \qquad (11)$$
$$I_i(i\tau) = q_{i_v}. \quad (12)$$

Solving (10) with the help of (11), we get

$$I_i(t) = \frac{ka}{b}\left[e^{bt} - e^{b((i-1)\tau+t_r)}\right] ; i = 1, 2, 3, \ldots, n \qquad (13)$$

Now, from (12) and (13), we get

$$q_{i_v} = \frac{ka}{b}\left[e^{bi\tau} - e^{b((i-1)\tau+t_r)}\right] ; i = 1, 2, 3, \ldots, n \qquad (14)$$

Therefore, the total ordered quantity Q produced by the vendor is given by

$$Q = \frac{ak}{b}\left[\frac{\{e^{b(n+1)\tau} - e^{b\tau}\} - e^{bt_r}\{e^{bn\tau} - 1\}}{e^{b\tau} - 1}\right] \qquad (15)$$

The total inventory carried out by the vendor through the ith interval is given by

$$I_{i_v} = \int_{(i-1)\tau+t_r}^{i\tau} I_i(t)dt = A_2e^{bi\tau} ; i = 1, 2, 3, \ldots, n \qquad (16)$$

Where $\qquad A_2 = \frac{ka}{b}\left[\frac{1 - e^{b(t_r-\tau)}}{b} + (t_r - \tau)e^{b(t_r-\tau)}\right]$.

Therefore, the total inventory carried out by the vendor through the entire cycle is given by

$$I_v = \sum_{i=1}^{n} I_{i_v} = A_2\left[\frac{e^{b(n+1)\tau} - e^{b\tau}}{e^{b\tau} - 1}\right] = A_2R$$

Therefore, the total cost per unit time for the vendor is given by

$$ATC_v(n, \tau) = \frac{C_0}{n\tau} + \frac{h_v}{n\tau}A_2R \qquad (17)$$

Integrated perspective:

In this case, we have $q_{i_b} = q_{i_v}$ and therefore, from (6) and (14), we get

$$t_r = \frac{1}{b} \log \left[\frac{(k+1)e^{b\tau} - e^{2b\tau}}{k} \right] \qquad (18)$$

Then equation (16) reduces to

$$I_{i_v} = \int_{(i-1)\tau+t_r}^{i\tau} I_i(t)dt = A_3 e^{bi\tau} \; ; i = 1, 2, 3, \dots, n \qquad (19)$$

where $A_3 = \frac{a}{b^2}[e^{b\tau} - 1] + \frac{a}{b^2}(k + 1 - e^{b\tau}) \log \left[\frac{k+1-e^{b\tau}}{k} \right]$

Therefore, the total cost per unit time for integrated supply chain is given by

$$ATC(n, \tau) = \frac{C_0 + C_1 + nC_2}{n\tau} + \frac{(A_1 h_b + A_3 h_v)}{n\tau} R \qquad (21)$$

Our objective is to determine the optimal number of shipments n^* and the optimal shipment quantity q_i^* for the ith ($i = 1, 2, \dots n$) shipment so that the average cost of the supply chain is minimized. In the objective function ATC, n is integer and τ is real. Hence, differentiating (21) with respect to τ we get

$$\frac{d}{d\tau}(ATC) = -\frac{1}{n\tau^2}\{(C_0 + C_1 + nC_2) + (A_1 h_b + A_3 h_v)R\} + \frac{1}{n\tau}\left\{(A_1 h_b + A_3 h_v)\frac{dR}{d\tau} + R\left(h_b \frac{dA_1}{d\tau} + h_v \frac{dA_3}{d\tau}\right)\right\}$$

where

$$\frac{dR}{d\tau} = \left[\frac{bn e^{b(n+2)\tau} - b(n+1)e^{b(n+1)\tau} + b e^{b\tau}}{(e^{b\tau} - 1)^2} \right] \qquad (22)$$

$$\frac{dA_1}{d\tau} = a\tau e^{b\tau} \qquad (23)$$

$$\frac{dA_3}{d\tau} = \frac{a}{b} e^{b\tau} \log \left[\frac{k}{k+1-e^{b\tau}} \right] \qquad (24)$$

Now, for optimality, $\frac{d}{d\tau}(ATC) = 0$ gives

$$\left\{(A_1 h_b + A_3 h_v)\frac{dR}{d\tau} + R\left(h_b \frac{dA_1}{d\tau} + h_v \frac{dA_3}{d\tau}\right)\right\}\tau - \{(C_0 + C_1 + nC_2) + (A_1 h_b + A_3 h_v)R\} = 0 \qquad (25)$$

Theorem 1: The integrated average total cost ATC increases as k increases.

Proof: We have

$$ATC = \frac{C_0 + C_1 + nC_2}{n\tau} + \frac{(A_1 h_b + A_3 h_v)}{n\tau} R$$

where

$$A_1 = -\frac{a}{b^2}[e^{b\tau} - 1] + \frac{a\tau}{b}e^{b\tau}$$

$$A_3 = \frac{a}{b^2}[e^{b\tau} - 1] + \frac{a}{b^2}(k + 1 - e^{b\tau}) \log \left[\frac{k+1-e^{b\tau}}{k} \right]$$

$$R = \frac{e^{b(n+1)\tau} - e^{b\tau}}{e^{b\tau} - 1}$$

Let $c = 1 - e^{b\tau}$. Then $c < 0$ as $b > 0$ and $\tau > 0$.

Let k_1 and k_2 be two values of k such that $k_1 < k_2$. Then

$$ATC(k_1) - ATC(k_2) = \frac{a}{b^2}\left[c \log \left\{ \frac{k_2(k_1 + c)}{k_1(k_2 + c)} \right\} + \log \left\{ \frac{\left(\frac{k_1+c}{k_1}\right)^{k_1}}{\left(\frac{k_2+c}{k_2}\right)^{k_2}} \right\} \right]$$

Now $k_1 < k_2 \Rightarrow ck_1 > ck_2 \Rightarrow k_1 k_2 + ck_1 > k_1 k_2 + ck_2 \Rightarrow k_1(k_2 + c) > k_2(k_1 + c) \Rightarrow \frac{k_2(k_1+c)}{k_1(k_2+c)} > 1$.

Therefore, $c \log \left\{ \frac{k_2(k_1+c)}{k_1(k_2+c)} \right\} < 0$.

Again $k_1 < k_2 \Rightarrow \frac{1}{k_1} > \frac{1}{k_2} \Rightarrow \frac{c}{k_1} < \frac{c}{k_2} \Rightarrow 1 + \frac{c}{k_1} < 1 + \frac{c}{k_2} \Rightarrow \frac{k_1+c}{k_1} < \frac{k_2+c}{k_2} \Rightarrow \left(\frac{k_1+c}{k_1}\right)^{k_1} < \left(\frac{k_2+c}{k_2}\right)^{k_2}$

Therefore, $\log \left\{ \frac{\left(\frac{k_1+c}{k_1}\right)^{k_1}}{\left(\frac{k_2+c}{k_2}\right)^{k_2}} \right\} < 0$.

Hence $ATC(k_1) - ATC(k_2) < 0$, i.e., $ATC(k_1) < ATC(k_2)$.

Theorem 2: *The total order quantity Q increases as a increases.*

Proof: Let a_1 and a_2 be two values of a such that $a_1 < a_2$. Then

$Q(a_1) - Q(a_2) = \frac{1}{b}\left(e^{b(n+1)\tau} - e^{b\tau}\right)(a_1 - a_2)$

Now, $n + 1 > 1 \Rightarrow e^{b(n+1)\tau} > e^{b\tau}$, since $b > 0$ and $\tau > 0$.

Therefore, $\frac{1}{b}\left(e^{b(n+1)\tau} - e^{b\tau}\right) > 0$ and hence $Q(a_1) < Q(a_2)$.

Theorem 3: *The value of t_r increases as k increases.*

Proof: Let k_1 and k_2 be two values of k such that $k_1 < k_2$. Then

$t_r(k_1) - t_r(k_2) = \frac{1}{b}\log\left\{\frac{k_2(k_1 + 1 - e^{b\tau})}{k_1(k_2 + 1 - e^{b\tau})}\right\}$

Let $c = 1 - e^{b\tau}$. Then $c < 0$ as $b > 0$ and $\tau > 0$ and therefore,

$t_r(k_1) - t_r(k_2) = \frac{1}{b}\log\left\{\frac{k_2(k_1 + c)}{k_1(k_2 + c)}\right\} < 0$, using Theorem 1.

Thus the theorem holds.

FUZZY PRELIMINARIES:

In this section, we provide the necessary background and some notions of fuzzy set theory.

Definition 1:

Let X denote a universal set. Then the fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x): X \to [0,1]$ which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0, 1]$ to each element $x\epsilon X$ and $\mu_{\tilde{A}}(x)$ shows the grade of membership of $x\epsilon\tilde{A}$.

Definition 2:

A triangular fuzzy number (TFN) $\tilde{A}$ can be denoted by three real numbers l, m and u where the parameters l, m and u denote the smallest, the most promising and the largest possible values respectively. The membership function of TFN can be expressed as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ \frac{u-x}{u-m}, & m \le x \le u \\ 0, & \text{otherwise} \end{cases};$$

A graphical representation of $\mu_{\tilde{A}}(x)$ is shown in Figure 2.

Figure 2: The fuzzy membership function for TFN
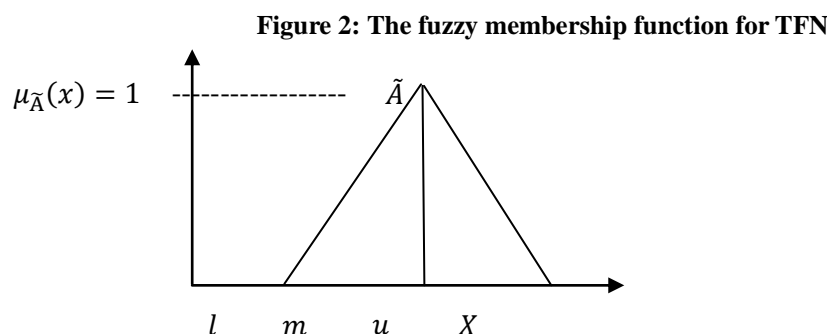
Let $\tilde{A} = (l_1, m_1, u_1)$, $\tilde{B} = (l_2, m_2, u_2)$ be two TFNs, then $\tilde{A} + \tilde{B} = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$

Centroid Method:

Defuzzification of $\tilde{A}$ can be done by applying centroid method. If $\tilde{A}$ is a TFN and fully determined by (l, m, u) then the centroid (A^*) of $\tilde{A}$ is given by

$A^* = \frac{\int \mu_{\tilde{A}}(A)A dA}{\int \mu_{\tilde{A}}(A)dA} = \frac{l+m+u}{3}$, where $\mu_{\tilde{A}}(m) = 1, \mu_{\tilde{A}}(l) = 0 = \mu_{\tilde{A}}(u)$.

FUZZY MODEL:

We consider all the cost components in our model in fuzzy sense. We represent set up cost, ordering cost, transportation cost, holding costs of the vendor and the buyer by symmetric TFNs as follows:

$\tilde{C}_o = (C_o - \delta_0 , C_o, C_o + \delta_0)$; $\tilde{C}_1 = (C_1 - \delta_1, C_1, C_1 + \delta_1)$; $\tilde{C}_2 = (C_2 - \delta_2, C_2, C_2 + \delta_2)$

$\tilde{h}_b = (h_b - \delta_3 , h_b, h_b + \delta_3)$; $\tilde{h}_v = (h_v - \delta_4 , h_v, h_v + \delta_4)$

where, C_o, C_1, C_2, h_b, h_v are the most promising values of $\tilde{C}_o, \tilde{C}_1, \tilde{C}_2, \tilde{h}_b$ and $\tilde{h}_v$ respectively. δ_i's are arbitrary positive numbers with the following restrictions:

$C_o > \delta_0, C_1 > \delta_1, C_2 > \delta_2, h_b > \delta_3, h_v > \delta_4$.

Using fuzzy costs, equations (21) and (25) can be written as

$$\widetilde{ATC} = \frac{\tilde{C}_o + \tilde{C}_1 + n\tilde{C}_2}{n\tau} + \frac{(A_1 \tilde{h}_b + A_3 \tilde{h}_v)}{n\tau}.R \qquad (26)$$

$$\left\{ \left(A_1 \tilde{h}_b + A_3 \tilde{h}_v \right)\frac{dR}{d\tau} + R\left(h_b \frac{dA_1}{d\tau} + h_v \frac{dA_3}{d\tau} \right) \right\}\tau - \left\{ \left(\tilde{C}_o + \tilde{C}_1 + n\tilde{C}_2 \right) + (A_1 \tilde{h}_b + A_3 \tilde{h}_v)R \right\} = 0 \qquad (27)$$

For a known value of n, equation (27) can be solved by any one dimensional search method.

NUMERICAL STUDY:

We suppose that the vendor and the buyer estimate the cost parameters within some ranges such as set up cost in between $240 and $260, ordering cost in between $190 and $210, transportation cost is around $90 to $110 per shipment, holding cost per unit per day for the buyer is in between $1.5 and $2.5 and that for the vendor is in between $3.5 to $4.5.

The triangular fuzzy numbers for the possible costs are:

$\tilde{C}_o = (240 , 250, 260)$, $\tilde{C}_1 = (190,200,210)$, $\tilde{C}_2 = (90,100,110)$, $\tilde{h}_b = (1.5 , 2, 2.5)$ and $\tilde{h}_v = (3.5 , 4, 4.5)$. Now to solve our problem, we consider $a = 500$, $b = 0.98$ and $k = 1.7$. The computational results obtained are shown in Table 1.

Table 1: Optimal results of the integrated vendor-buyer model for different values of n.

n	τ	Q	$ATC(n, Q)$
1	0.18364	120.44	2280.69
2	0.19299	283.39	2021.71
3	0.20083	499.91	1643.49
	0.20702	782.01	1495.64
5	0.21123	1139.16	1447.38
6	0.21311	1572.50	1452.04
7	0.21243	2069.62	1484.83

Table 1 shows that for the coordinated system, the optimal number of shipments is 5 and the optimal order quantity is 1139.16 units and the minimum average total cost is1447.38 units.

Non-coordinated Policy:
(a) Buyer's perspective:

From equation (9), the buyer's optimal ordered quantity is obtained as $Q_b^* = Q^* = 967.76$ units and the average total cost as $ATC_b^* = 856.17$ units. When the buyer's optimal ordered quantity $Q^* = 967.76$ is accepted by the vendor then from equation (17), the average total cost for the vendor becomes $ATC_v^* = 859.02$ units. Thus the combined average system cost becomes 1715.19 units which is significantly greater than (by a margin of 267.81 units) that obtained in the coordinated policy.

(b) Vendor's perspective:

When vendor minimizes his average total cost, we obtain then from equation (17), his optimal ordered quantity and the average total cost as $Q_v^* = Q^* = 3548.72$ units and $ATC_v^* = 562.95$ units, respectively. Using $Q^* = 3548.72$ in equation (9) the average total cost for the buyer becomes $ATC_b^* = 1178.23$ units. Therefore, the combined average total cost becomes 1741.18 units which is again significantly greater than (by a margin of 293.80 units) that obtained in the coordinated policy.

From the numerical study we can conclude that the coordination between vendor and buyer gives better result than the non-coordinated policy.

We now examine the sensitivity of the optimal results obtained in Table 1 to changes in the parameter-values of a, b and k. While computing, the value of a single parameter is changed from -50% to $+50\%$ and the other parameters are kept unchanged. The results are shown in Table 2.

Table 2: Effects of changes in the parameter values on the optimal results

Parameter	Percentage change in parameter value	Percentage change in	
		Q^*	ATC^*
a	+50	+53	+14
	+20	+45	+6
	−20	−6	−8
	−50	−16	−20
b	+50	+23	+17
	+20	+9	+7
	−20	+24	−7
	−50	+32	−19
k	+50	+28	+6
	+20	+12	+3
	−20	−7	−4
	−50	−13	−16

On the basis of the results shown in Table 2, the following observations can be made.
- Both Q^* and ATC^* increase or decrease as a increases or decreases.
- Any change in b results in the increment in Q^*.
- ATC^* is consistent with b and slightly less sensitive to the parameter k.
- Q^* is highly sensitive to the parameter k.

CONCLUSION:

In this paper, we have developed a vendor-buyer model implementing fuzzy set theory which makes the model more realistic. We have used symmetric TFNs to represent the imprecise set up cost, ordering cost, transportation cost and holding cost and obtained the model with fuzzy total cost. For defuzzification of the fuzzy total cost, we have applied centroid method. Our model is so developed that shortages can be avoided at the buyer's end. We have also considered that the market demand is known to both the buyer and vendor and accordingly the production time is set by the vendor so that there remains hardly any excess inventory in either one's stock. Moreover, there is a relaxation of delaying to set the production starting point at the vendor's side.
It is observed from the numerical study that the average cost for the coordinated policy is lesser than that obtained in the non-coordinated policy. This again established that coordination between vendor and buyer is very much essential to obtain better optimal results.
Future researchers may extend this work by considering the other parameters as fuzzy and different defuzzification methods may be adopted to obtain the optimal results in crisp. One may consider imperfect production system and shortages in buyer's inventory for the extension of this work.

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