A Single-Vendor Single-Buyer Supply Chain Coordination Model with Price Discount and Benefit Sharing in Fuzzy Environment

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ABSTRACT

Coordination among the participating members of a supply chain is a major issue for all successful decision makers. In the literature, numerous supply chain models have been developed based on exact (known) parameter-values. However, in reality, vagueness of parameter-values is frequently observed in many circumstances. In this article, we develop a two-stage supply chain model with a single vendor and a single buyer, and design a coordination mechanism through price discount policy with incomplete information of demand and cost parameters. We first develop the model imposing fuzziness in demand and then fuzziness in all the cost components. In each case, centroid method is used for defuzzification. A solution procedure is outlined and suitable numerical examples are given to determine the optimal results of the proposed fuzzy models.

Keywords: supply chain management, coordination, price discount, fuzzy, centroid method.

INTRODUCTION:

The single-vendor single-buyer integrated production-inventory problem has received a maximum attention from both academia and industry in recent years due to the growing focus on supply chain management. Researchers have shown their keen interest on supply chain management realizing its potential to improve performance of business firms at a reduced cost and delivery time. Moreover, an efficient management of inventories across the entire supply chain through better coordination and cooperation can give better joint benefit to all members involved. This leads the researchers to develop the strategic coordination between vendor and buyer of a supply chain.

One of the first works dealing with integrated vendor-buyer supply chain is due to Goyal (1976). In his work, he addressed a simple supply chain model with single supplier and single customer problem. Later, Banerjee (1988) developed a joint economic lot size model assuming that the vendor manufactures at a finite rate. He considered a lot-for-lot model where the vendor produces each buyer shipment as a separate batch. Goyal (1988) argued this assumption and proposed that producing a batch which is made up of equal shipments provides lower cost. Lu (1995) developed a single vendor single buyer model with equal shipments and derived its optimal solutions. In another work, Goyal (1995) showed that different shipment size policy could produce a better solution. He considered successive shipments within a production batch increased by a constant factor which is equal to the ratio of production rate over the demand rate. Hill (1999) proposed a globally optimal batching and shipping policy for single vendor single buyer supply chain. Considering unequal and equal shipments from the vendor to the buyer, Hogue and Goyal (2000) derived a solution procedure for optimal

production quantity. They considered capacity constraint of the transport equipment in their model. Ben-Daya and Hariga (2004) used stochastic demand and variable lead time in their model and showed that coordination is effective from the vendor's as well as the buyer's perspectives. Considering probabilistic demand and using quantity discount policy, Li and Liu (2006) proposed a model to achieve coordination between the vendor and the buyer. Qin et al. (2007) developed a single vendor single buyer integrated inventory model considering volume discount, franchise fees and price sensitive demand. Sajadieh et al. (2009) proposed another single vendor single buyer supply chain model in which the vendor delivers the production batch to the buyer in n equal shipments and the lead time is stochastic. Yang (2010) contributed, in his supply chain model, on the present value analysis assuming variable lead time. Masihabadi and Eshghi (2011) developed a model for coordinating seller-buyer with a proper allocation of chain's surplus profit assuming a general side payment contract. Shahrjerdi et al. (2011) proposed a cooperative and non-cooperative seller-buyer supply chain model and derived the optimal decisions. Saha et al. (2012) studied a supply chain model considering stock and price dependent selling rates in the declining market. Moharana et al. (2012) developed a supply chain model where they investigated on the stability of coordination, collaboration and integration between the buyer and the vendor. Shah et al. (2013) proposed a single-supplier single-buyer inventory system to derive the optimal pricing, shipments and ordering policies. They assumed the demand as price-sensitive and stock dependent and also the order-linked trade-credit. Dey and Giri (2014) proposed a vendor-buyer model under imperfect production; Zhang et al. (2016) considered discount policy to develop their model; Li et al. (2017) investigated the environmental impacts in their model.

In the above studies, researchers used crisp values for the cost parameters. However, in reality, these costs may not be known as fixed quantities. For example, set up cost and holding cost of the vendor may vary due to some changes in environment. A retailer may face the same problem to determine the replenishment cost. A lot of probabilistic models have been developed to capture various uncertainties in reality. Researchers have used statistical methods to estimate the parameter(s) of probability distribution from the past data. However, past data may not be always available or reliable. To cope with this problem, one can use possibility theory rather than probability theory and represent uncertainty by possibility distribution.

Zadeh (1965) was first to introduce fuzzy set theory to handle these vague situations. In recent years, the theory of fuzzy sets and fuzzy logic has found wide applications in the field of operations management. Yao and Lee (1996) proposed a back order fuzzy inventory model with fuzzy order quantity as triangular and trapezoidal fuzzy numbers and shortage cost as a crisp parameter. Yao and Chiang (2003), in their inventory model without backorder, fuzzified the total demand and cost of storing by triangular fuzzy numbers and defuzzified the total cost by centroid and signed distance methods. Chiang et al. (2005) represented the storing cost, backorder cost, ordering cost, total demand, order quantity and shortage quantity by triangular fuzzy numbers and defuzzified by signed distance method. Tutunchu and Akoz (2008) used fuzzy setup cost, holding cost and shortage cost, and defuzzified by Park's Median Rule. Lin (2008) fuzzified the expected demand, shortage and backorder rate and used signed distance method to defuzzify the cost function obtained in the fuzzy sense. Sadi-Nezhad et al. (2011) in their continuous review inventory model fuzzified the setup cost, holding cost and shortage cost and defuzzified by signed distance and possibilistic mean value methods. Lee and Lin (2011), Sarkar and Chakrabarti (2012), Mahapatra et al. (2012) proposed some fuzzy inventory models.

The literature shows that there has been a significant development of inventory models under uncertainty using the concept of fuzzy sets. However, study of supply chain model in fuzzy environment is limited. Mahata et al. (2005) proposed a joint economic lot size model for both buyer and vendor assuming the order quantity as a fuzzy variable and other parameters as deterministic. Corbett and Groote (2000) developed a supply chain model under asymmetric information where they considered quantity discount policy. Petrovic et al. (1999) used fuzzy set theory in their supply chain model. Gunasekaran et al. (2006) contributed in optimizing the order quantity with fuzzy approach. Eric Sucky (2006) proposed a two-staged supply chain coordination problem under asymmetric information where the assumption is that the buyers have two sets of predefined (deterministic) cost structures and the vendor does not know in which particular cluster a particular buyer belongs. However, this model has a limitation to address a wide range of cost structures that a buyer can possibly have. Ganga and Carpinetti (2011) and Costantino et al. (2012) proposed some valuable supply chain models in fuzzy environment. Sinha and Sarmah (2008) extended their work considering quantity discount policy. They assumed ordering cost, inventory holding cost of the buyer and demand as fuzzy variables. Taleizadeh et al. (2013) considered fuzzy-lead times and suggested two hybrid procedures to solve the integrated inventory problems. Mahata (2015) developed a single-vendor single-buyer model with fuzzy order quantity and fuzzy shortage quantity. Priyam and Uthayakumar (2016) proposed a fuzzy model under variable

lead time and service level constraint. Jauhari and Saga (2017) considered set up cost reduction and service level constraint in their vendor-buyer model.

In this article, we consider the demand at the buyer and all the cost components of the vendor and buyer as fuzzy variables. We use centroid method for defuzzification of the average fuzzy cost. The rest of the paper is organized as follows: In Section 2, some assumptions and notations are given. In Section 3, the mathematical formulation of the model is given. Section 4 expresses some preliminaries on fuzzy set theory and defuzzification method. Section 5 deals with the implementation of fuzzy concept in the inventory system. Numerical examples are given in Section 6 to illustrate the developed models and perform sensitivity analysis with respect to some model-parameters. Finally, we conclude the paper in Section 7.

ASSUMPTIONS AND NOTATIONS:

The following assumptions are made for developing the proposed model:
(i)   A single-type of product is considered.
(ii)   The supply chain consists of a single buyer and a single vendor.
(iii)   The vendor's production rate is finite and is greater than the buyer's demand rate.
(iv)   The vendor offers price discount to the buyer.
(v)   Each production lot size of the vendor is transported to buyer in n equal shipments.
(vi)   The vendor's production process is reliable (perfect).
(vii)   Shortages are not allowed in buyer's inventory.

The following notations are used to construct our mathematical model:

$D\ (\widetilde{D})$: Demand rate in units per unit time in crisp (fuzzy).
$\frac{1}{p}$: Production rate in units per unit time.
n : Number of shipments from the vendor to the buyer.
Q : Size of equal shipments from the vendor to the buyer.
$C_0\ (\tilde{C}_o)$: Vendor's set up cost in crisp (fuzzy).
$C_1\ (\tilde{C}_1)$: Buyer's ordering cost for each order of size nQ in crisp (fuzzy).
$C_2\ (\tilde{C}_2)$: Buyer's transportation cost for each shipment of size Q in crisp (fuzzy).
$C_3\ (\tilde{C}_3)$: Vendor's unit production cost in crisp (fuzzy).
$C_0\ (\tilde{C}_o)$: Vendor's undiscounted sale price in crisp (fuzzy).
C_5 : Vendor's discounted sale price in crisp.
$C_6\ (\tilde{C}_6)$: Buyer's unit sale price in crisp.
$h_b\ (\tilde{h}_b)$: Buyer's holding cost per unit per time in crisp (fuzzy).
$h_v\ (\tilde{h}_v)$: Vendor's holding cost per unit per time for the vendor in crisp (fuzzy).
λ : Level of benefit offered by the vendor to the buyer.

MATHEMATICAL FORMULATION:

The proposed single-vendor single-buyer supply chain works as follows. The vendor obtains an order of size nQ from the buyer. The vendor starts his production with a finite and uniform production rate which is greater than the demand rate of the buyer. The buyer receives n lots of size Q in n shipments. The inventory profiles for the buyer and vendor are depicted in Figs. 1 and 2, respectively.
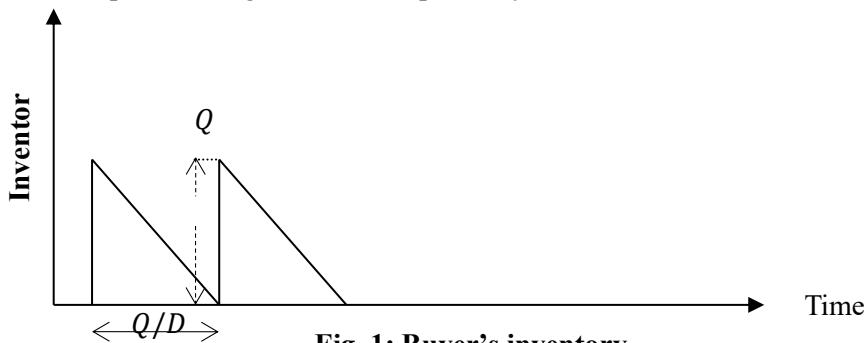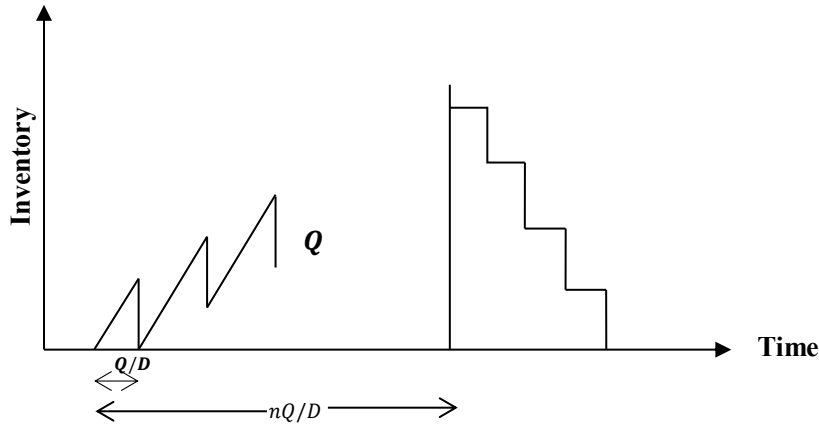

Fig. 1: Buyer's inventory

Fig.2: Vendor's inventory

Non-coordinated policy:

In this section, we discuss the non-coordinated policy from buyer's and vendor's perspectives separately.

Buyer's perspective:

The total cost per unit time for the buyer is given by

$$\left(\frac{C_1}{n} + C_2\right).\frac{D}{Q} + h_b\frac{Q}{2}$$

The average profit of the buyer is given by

$$BP_{nc} = (C_6 - C_4)D - \left[\left(\frac{C_1}{n} + C_2\right)\frac{D}{Q} + h_b\frac{Q}{2}\right] \tag{1}$$

The buyer can determine the economic order quantity which maximizes BP_{nc} by using the first order condition for optimality $\frac{dBP_{nc}}{dQ} = 0$ which gives

$$Q_b^* = \sqrt{\frac{2\left(\frac{C_1}{n} + C_2\right)D}{h_b}} \tag{2}$$

And thus

$$BP_{nc}^* = (BP_{nc})_{max} = (C_6 - C_4)D - \left[\left(\frac{C_1}{n} + C_2\right)\frac{D}{Q_b^*} + h_b\frac{Q_b^*}{2}\right] \tag{3}$$

On the other hand, when the buyer's economic order quantity (Q_b^*) is accepted by the vendor, the total cost per unit time for the vendor is given by

$$\frac{C_0 D}{n Q_b^*} + h_v\{n(1 - Dp) - 1 + 2Dp\}\frac{Q_b^*}{2}$$

Therefore, the average of the vendor is given by

$$VP_{nc}^* = (C_4 - C_3)D - \left[\frac{C_0 D}{n Q_b^*} + h_v\{n(1 - Dp) - 1 + 2Dp\}\frac{Q_b^*}{2}\right] \tag{4}$$

Therefore, from equations (3) and (4), we get the average profit of the system as

$$AP_{nc}^* = BP_{nc}^* + VP_{nc}^* = (C_6 - C_3)D - \left[\left(\frac{C_0 + C_1}{n} + C_2\right)\frac{D}{Q_b^*} + (h_b + h_v\{n(1 - Dp) - 1 + 2Dp\})\frac{Q_b^*}{2}\right] \tag{5}$$

Vendor's perspective:

The average profit of the vendor is given by

$$V_{nc} = (C_4 - C_3)D - \left[\frac{C_0 D}{nQ} + h_v\{n(1 - Dp) - 1 + 2Dp\}\frac{Q}{2}\right] \tag{6}$$

Using the first order condition, the optimal value of Q can be obtained as

$$Q_v^* = \sqrt{\frac{2\left(\frac{C_0}{n}\right)D}{h_v\{n(1 - Dp) - 1 + 2Dp\}}} \tag{7}$$

Then

$$VP_{nc}^* = (VP_{nc})_{max} = (C_4 - C_3)D - \left[\frac{C_0 D}{nQ_v^*} + h_v\{n(1 - Dp) - 1 + 2Dp\}\frac{Q_v^*}{2}\right] \tag{8}$$

When Q_v^* is accepted by the buyer, the average profit of the buyer is given by

$$BP_{nc}^* = (C_6 - C_4)D - \left[\left(\frac{C_1}{n} + C_2\right)\frac{D}{Q_v^*} + h_b \frac{Q_v^*}{2}\right] \tag{9}$$

Therefore, from equations (8) and (9), we get the average profit of the system as

$$AP_{nc}^* = VP_{nc}^* + BP_{nc}^* = (C_6 - C_3)D - \left[\left(\frac{C_0+C_1}{n} + C_2\right)\frac{D}{Q_v^*} + (h_b + h_v\{n(1-Dp) - 1 + 2Dp\})\frac{Q_v^*}{2}\right] \tag{10}$$

Coordinated policy:

When the vendor's discounted sale price is C_5 per unit then the average profit of the buyer is given by

$$BP_c = (C_6 - C_5)D - \left[\left(\frac{C_1}{n} + C_2\right)\frac{D}{Q} + h_b \frac{Q}{2}\right] \tag{11}$$

The average profit of the vendor is given by

$$VP_c = (C_5 - C_3)D - \left[\frac{C_0 D}{nQ} + h_v\{n(1-Dp) - 1 + 2Dp\}\frac{Q}{2}\right] \tag{12}$$

From equations (11) and (12), the average co-ordinated profit of the supply chain is given by

$$AP_c = BP_c + VP_c = (C_6 - C_3)D - \left[\left(\frac{C_0+C_1}{n} + C_2\right)\frac{D}{Q} + (h_b + h_v\{n(1-Dp) - 1 + 2Dp\})\frac{Q}{2}\right] \tag{13}$$

Our objective is to determine the optimal number of shipments n* and the shipment size Q* so that the average co-ordination profit AP_c is maximized. The first order optimality condition $\frac{dAP_c}{dQ} = 0$ gives

$$Q^* = \sqrt{\frac{2\left(\frac{C_0+C_1}{n}+C_2\right)D}{h_b+h_v\{n(1-Dp)-1+2Dp\}}} \tag{14}$$

Now, benefit of the buyer if price discount is implemented, is given by

$$BP_b = (BP_c - BP_{nc}^*) = \begin{cases} (C_4 - C_5)D + \left(\frac{C_1}{n} + C_2\right)\left(\frac{1}{Q_b^*} - \frac{1}{Q^*}\right)D + \frac{h_b}{2}(Q_b^* - Q^*) \\ (C_4 - C_5)D + \left(\frac{C_1}{n} + C_2\right)\left(\frac{1}{Q_v^*} - \frac{1}{Q^*}\right)D + \frac{h_b}{2}(Q_v^* - Q^*) \end{cases} \tag{15.a}$$

and benefit of the vendor is given by

$$BP_v = (VP_c - VP_{nc}^*) = \begin{cases} (C_5 - C_4)D + \frac{C_0 D}{n}\left(\frac{1}{Q_b^*} - \frac{1}{Q^*}\right) + \frac{h_v}{2}(Q_b^* - Q^*)\{n(1-Dp) - 1 + 2Dp\} \\ (C_5 - C_4)D + \frac{C_0 D}{n}\left(\frac{1}{Q_v^*} - \frac{1}{Q^*}\right) + \frac{h_v}{2}(Q_v^* - Q^*)\{n(1-Dp) - 1 + 2Dp\} \end{cases} \tag{15.b}$$

It is assumed that the vendor offers the buyer a certain level of co-ordination benefit such that

$$BP_b = \lambda BP_v, \lambda \geq 0 \tag{15.c}$$

Therefore, from equations (15.a), (15.b) and (15.c), we get

$$C_5 = \begin{cases} C_4 + \frac{\left(\frac{1}{Q_b^*} - \frac{1}{Q^*}\right)}{(1+\lambda)}\left(\frac{C_1-\lambda C_0}{n} + C_2\right) + \frac{(Q_b^* - Q^*)}{2D(1+\lambda)}[h_b - \lambda h_v\{n(1-Dp) - 1 + 2Dp\}] \\ C_4 + \frac{\left(\frac{1}{Q_v^*} - \frac{1}{Q^*}\right)}{(1+\lambda)}\left(\frac{C_1-\lambda C_0}{n} + C_2\right) + \frac{(Q_v^* - Q^*)}{2D(1+\lambda)}[h_b - \lambda h_v\{n(1-Dp) - 1 + 2Dp\}] \end{cases} \tag{16}$$

Theorem 1: The average joint profit AP_c is concave with respect to Q.
Proof: We have from (13),

$$AP_c = (C_6 - C_3)D - \left[\left(\frac{C_0+C_1}{n} + C_2\right)\frac{D}{Q} + (h_b + h_v\{n(1-Dp) - 1 + 2Dp\})\frac{Q}{2}\right]$$

Therefore, $\frac{dAP_c}{dQ} = \left(\frac{C_0+C_1}{n} + C_2\right)\frac{D}{Q^2} - (h_b + h_v\{n(1-Dp) - 1 + 2Dp\})\frac{1}{2}$

and

$$\frac{d^2 APT_c}{dQ^2} = -2\left(\frac{C_0+C_1}{n} + C_2\right)\frac{D}{Q^3}$$

Since $C_0 > 0, C_1 > 0, C_2 > 0, D > 0, n > 0$ and $Q > 0$, we get $\frac{d^2 AP_c}{dQ^2} < 0$.

Thus the average joint profit AP_c is a concave function in Q.

FUZZY PRELIMINARIES:

In this section, the necessary background and some notions of fuzzy set theory are given.
Definition 1:
Let X denote a universal set. Then the fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x): X \rightarrow$

[0,1] which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$ to each element $x \epsilon X$ and $\mu_{\tilde{A}}(x)$ shows the grade of membership of $x \epsilon \tilde{A}$.

Definition 2:

A triangular fuzzy number (TFN) $\tilde{A}$ can be denoted by three real numbers l, m and u where the parameters l, m and u denote the smallest, the most promising and the largest possible values respectively. The membership function of TFN can be expressed as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l} , l \leq x \leq m \\ \frac{u-x}{u-m} , m \leq x \leq u \\ 0 , \text{otherwise} \end{cases} ;$$

A graphical representation of $\mu_{\tilde{A}}(x)$ is shown in Fig. 3.
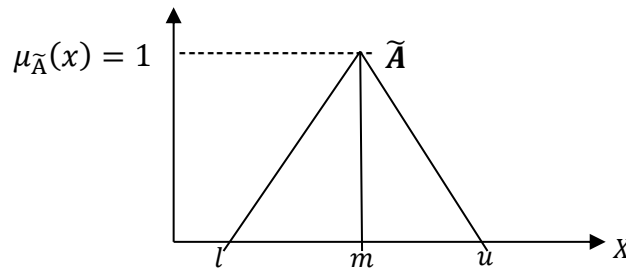

Fig. 3 The fuzzy membership function for TFN

Let $\tilde{A} = (l_1, m_1, u_1), \tilde{B} = (l_2, m_2, u_2)$ be two TFNs, then
$\tilde{A} + \tilde{B} = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$

Centroid method:

If $\tilde{A}$ is a TFN and is fully determined by (l, m, u) then the centroid (A) of $\tilde{A}$ is given by

$$A = \frac{\int \mu_{\tilde{A}}(A) A dA}{\int \mu_{\tilde{A}}(A) dA} = \frac{(l+m+n)}{3} , \text{where } \mu_{\tilde{A}}(m) = 1, \mu_{\tilde{A}}(l) = 0 = \mu_{\tilde{A}}(u).$$

FUZZY MODEL:

Since evaluating actual values of the parameters in real world applications is very difficult, we consider first the annual demand as fuzzy while the other parameter values are in crisp. We then assume all the costs components in fuzzy sense keeping the annual demand and other parameters in crisp. We represent annual demand, set up cost, ordering cost, transportation cost, per unit production cost, un-discounted per unit sell price of the vendor, per unit sell price of the buyer, holding cost per unit per unit time for the vendor, holding cost per unit per unit time for the buyer by symmetric TFNs as follows:

$\tilde{C}_0 = (C_0 - \delta_0, C_0, C_0 + \delta_0)$ $\tilde{C}_3 = (C_3 - \delta_3, C_3, C_3 + \delta_3)$ $\tilde{h}_b = (h_b - \delta_7, h_b, h_b + \delta_7)$
$\tilde{C}_1 = (C_1 - \delta_1, C_1, C_1 + \delta_1)$ $\tilde{C}_4 = (C_4 - \delta_4, C_4, C_4 + \delta_4)$ $\tilde{h}_v = (h_v - \delta_8, h_v, h_v + \delta_8)$
$\tilde{C}_2 = (C_2 - \delta_2, C_2, C_2 + \delta_2)$ $\tilde{C}_6 = (C_6 - \delta_6, C_6, C_6 + \delta_6)$ $\tilde{D} = (D - \delta_9, D, D + \delta_9)$

where, $C_0, C_1, C_2, C_3, C_4, C_6, h_b, h_v$ and D are the most promising values of $\tilde{C}_0, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3, \tilde{C}_4, \tilde{C}_6, \tilde{h}_b, \tilde{h}_v$ and $\tilde{D}$ respectively. δ_i's are arbitrary positive numbers with the following restrictions:
$C_o > \delta_0, C_1 > \delta_1, C_2 > \delta_2, C_3 > \delta_3, C_4 > \delta_4, C_6 > \delta_6, h_b > \delta_7, h_v > \delta_8, D > \delta_9$.

MODEL WITH FUZZY DEMAND:

Non-coordinated policy in fuzzy sense:

Buyer's perspective:

Using fuzzy demand, equations (2), (3), (4) and (5) can be written as follows:

$$\tilde{Q}_b^* = \sqrt{\frac{2\left(\frac{C_1}{n}+C_2\right)\tilde{D}}{h_b}} \qquad (17.a)$$

$$\widetilde{BP}_{nc}^* = (C_6 - C_4)\tilde{D} - \left[\left(\frac{C_1}{n} + C_2\right)\frac{\tilde{D}}{\tilde{Q}_b^*} + h_b\frac{\tilde{Q}_b^*}{2}\right] \qquad (17.b)$$

$$\widetilde{VP}_{nc}^* = (C_4 - C_3)\tilde{D} - \left[\frac{C_0\tilde{D}}{n\tilde{Q}_b^*} + h_v\{n(1 - \tilde{D}p) - 1 + 2\tilde{D}p\}\frac{\tilde{Q}_b^*}{2}\right] \qquad (17.c)$$

$$\widetilde{AP}_{nc}^* = (C_6 - C_3)\tilde{D} - \left[\left(\frac{C_0+C_1}{n} + C_2\right)\frac{\tilde{D}}{\tilde{Q}_b^*} + \left(h_b + h_v\{n(1 - \tilde{D}p) - 1 + 2\tilde{D}p\}\right)\frac{\tilde{Q}_b^*}{2}\right] \qquad (17.d)$$

Vendor's perspective:

After imposing fuzziness on demand equations (7), (8), (9) and (10) give

$$\tilde{Q}_v^* = \sqrt{\frac{2\left(\frac{C_0}{n}\right)\tilde{D}}{h_v\{n(1-Dp)-1+2Dp\}}} \qquad (18.a)$$

$$\widetilde{VP}_{nc}^* = (C_4 - C_3)\tilde{D} - \left[\frac{C_0\tilde{D}}{n\tilde{Q}_v^*} + h_v\{n(1 - \tilde{D}p) - 1 + 2\tilde{D}p\}\frac{\tilde{Q}_v^*}{2}\right] \qquad (18.b)$$

$$\widetilde{BP}_{nc}^* = (C_6 - C_4)\tilde{D} - \left[\left(\frac{C_1}{n} + C_2\right)\frac{\tilde{D}}{\tilde{Q}_v^*} + h_b\frac{\tilde{Q}_v^*}{2}\right] \qquad (18.c)$$

$$\widetilde{AP}_{nc}^* = (C_6 - C_3)\tilde{D} - \left[\left(\frac{C_0+C_1}{n} + C_2\right)\frac{\tilde{D}}{\tilde{Q}_v^*} + \left(h_b + h_v\{n(1 - \tilde{D}p) - 1 + 2\tilde{D}p\}\right)\frac{\tilde{Q}_v^*}{2}\right] \qquad (18.d)$$

Coordinated policy in fuzzy sense:

In fuzzy demand sense, equations (14), (11), (12), (13) and (16) take the forms as:

$$\tilde{Q}^* = \sqrt{\frac{2\left(\frac{C_0+C_1}{n}+C_2\right)\tilde{D}}{h_b+h_v\{n(1-\tilde{D}p)-1+2\tilde{D}p\}}} \qquad (19.a)$$

$$\tilde{C}_5^* = \begin{cases} C_4 + \frac{\left(\frac{1}{\tilde{Q}_b^*}-\frac{1}{Q^*}\right)}{(1+\lambda)}\left(\frac{C_1-\lambda C_0}{n} + C_2\right) + \frac{(\tilde{Q}_b^*-Q^*)}{2\tilde{D}(1+\lambda)}[h_b - \lambda h_v\{n(1-\tilde{D}p)-1+2\tilde{D}p\}] \\ C_4 + \frac{\left(\frac{1}{\tilde{Q}_v^*}-\frac{1}{Q^*}\right)}{(1+\lambda)}\left(\frac{C_1-\lambda C_0}{n} + C_2\right) + \frac{(\tilde{Q}_v^*-Q^*)}{2\tilde{D}(1+\lambda)}[h_b - \lambda h_v\{n(1-\tilde{D}p)-1+2\tilde{D}p\}] \end{cases} \qquad (19.b)$$

$$\widetilde{BP}_c^* = (C_6 - C_5)\tilde{D} - \left[\left(\frac{C_1}{n} + C_2\right)\frac{\tilde{D}}{Q} + h_b\frac{Q}{2}\right] \qquad (19.c)$$

$$\widetilde{VP}_c^* = (C_5 - C_3)D - \left[\frac{C_0\tilde{D}}{nQ} + h_v\{n(1 - \tilde{D}p) - 1 + 2\tilde{D}p\}\frac{Q}{2}\right] \qquad (19.d)$$

$$\widetilde{AP}_c^* = (C_6 - C_3)\tilde{D} - \left[\left(\frac{C_0+C_1}{n} + C_2\right)\frac{\tilde{D}}{\tilde{Q}^*} + \left(h_b + h_v\{n(1 - \tilde{D}p) - 1 + 2\tilde{D}p\}\right)\frac{\tilde{Q}^*}{2}\right] \qquad (19.e)$$

MODEL WITH FUZZY COST COMPONENTS:

Non-coordinated policy in fuzzy sense:
Buyer's perspective:
Using fuzzy costs, equations (2), (3), (4) and (5) become

$$\tilde{Q}_b^* = \sqrt{\frac{2\left(\frac{\tilde{C}_1}{n}+\tilde{C}_2\right)D}{\tilde{h}_b}} \qquad (20.a)$$

$$\widetilde{BP}_{nc}^* = (\tilde{C}_6 - \tilde{C}_4)D - \left[\left(\frac{\tilde{C}_1}{n} + \tilde{C}_2\right)\frac{D}{\tilde{Q}_b^*} + \tilde{h}_b\frac{\tilde{Q}_b^*}{2}\right] \qquad (20.b)$$

$$\widetilde{VP}_{nc}^* = (\tilde{C}_4 - \tilde{C}_3)D - \left[\frac{\tilde{C}_o D}{n\tilde{Q}_b^*} + \tilde{h}_v\{n(1 - Dp) - 1 + 2Dp\}\frac{\tilde{Q}_b^*}{2}\right] \qquad (20.c)$$

$$\widetilde{AP}_{nc}^* = (\tilde{C}_6 - \tilde{C}_3)D - \left[\left(\frac{\tilde{C}_0+\tilde{C}_1}{n} + \tilde{C}_2\right)\frac{D}{\tilde{Q}_b^*} + \left(\tilde{h}_b + \tilde{h}_v\{n(1 - Dp) - 1 + 2Dp\}\right)\frac{\tilde{Q}_b^*}{2}\right] \qquad (20.d)$$

Vendor's perspective:

Applying fuzzy costs from equations (7), (8), (9) and (10) we have

$$\tilde{Q}_v^* = \sqrt{\frac{2\left(\frac{\tilde{C}_o}{n}\right)D}{\tilde{h}_v\{n(1-Dp)-1+2Dp\}}} \qquad (21.a)$$

$$\widetilde{VP}_{nc}^* = (\tilde{C}_4 - \tilde{C}_3)D - \left[\frac{\tilde{C}_o D}{n\tilde{Q}_v^*} + \tilde{h}_v\{n(1-Dp)-1+2Dp\}\frac{\tilde{Q}_v^*}{2}\right] \qquad (21.b)$$

$$\widehat{BP}_{nc}^* = (\tilde{C}_6 - \tilde{C}_4)D - \left[\left(\frac{\tilde{C}_1}{n} + \tilde{C}_2\right)\frac{D}{\tilde{Q}_v^*} + \tilde{h}_b\frac{\tilde{Q}_v^*}{2}\right] \qquad (21.c)$$

$$\widetilde{AP}_{nc}^* = (\tilde{C}_6 - \tilde{C}_3)D - \left[\left(\frac{\tilde{C}_o + \tilde{C}_1}{n} + \tilde{C}_2\right)\frac{D}{\tilde{Q}_v^*} + \left(\tilde{h}_b + \tilde{h}_v\{n(1-Dp)-1+2Dp\}\right)\frac{\tilde{Q}_v^*}{2}\right] \qquad (21.d)$$

Coordinated policy in fuzzy sense

In fuzzy cost environment, equations (14), (11), (12), (13) and (16) give

$$\tilde{Q}^* = \sqrt{\frac{2\left(\frac{\tilde{C}_o + \tilde{C}_1}{n} + \tilde{C}_2\right)D}{\tilde{h}_b + \tilde{h}_v\{n(1-Dp)-1+2Dp\}}} \qquad (22.a)$$

$$\tilde{C}_5^* = \begin{cases} \tilde{C}_4 + \frac{\left(\frac{1}{\tilde{Q}_b^*} - \frac{1}{\tilde{Q}^*}\right)}{(1+\lambda)}\left(\frac{\tilde{C}_1 - \lambda\tilde{C}_o}{n} + \tilde{C}_2\right) + \frac{(\tilde{Q}_b^* - Q^*)}{2D(1+\lambda)}\left[\tilde{h}_b - \lambda\tilde{h}_v\{n(1-Dp)-1+2Dp\}\right] \\ \tilde{C}_4 + \frac{\left(\frac{1}{\tilde{Q}_v^*} - \frac{1}{\tilde{Q}^*}\right)}{(1+\lambda)}\left(\frac{\tilde{C}_1 - \lambda\tilde{C}_o}{n} + \tilde{C}_2\right) + \frac{(\tilde{Q}_v^* - Q^*)}{2D(1+\lambda)}\left[\tilde{h}_b - \lambda\tilde{h}_v\{n(1-Dp)-1+2Dp\}\right] \end{cases} \qquad (22.b)$$

$$\widehat{BP}_c^* = (\tilde{C}_6 - \tilde{C}_5)D - \left[\left(\frac{C_1}{n} + C_2\right)\frac{D}{Q} + h_b\frac{Q}{2}\right] \qquad (22.c)$$

$$\widetilde{VP}_c^* = (\tilde{C}_5 - \tilde{C}_3)D - \left[\frac{C_o D}{nQ} + h_v\{n(1-Dp)-1+2Dp\}\frac{Q}{2}\right] \qquad (22.d)$$

$$\widetilde{AP}_c^* = (\tilde{C}_6 - \tilde{C}_3)D - \left[\left(\frac{\tilde{C}_o + \tilde{C}_1}{n} + \tilde{C}_2\right)\frac{D}{\tilde{Q}^*} + \left(\tilde{h}_b + \tilde{h}_v\{n(1-Dp)-1+2Dp\}\right)\frac{\tilde{Q}^*}{2}\right] \qquad (22.e)$$

SOLUTION PROCEDURE:

To study our fuzzy model numerically, we have developed different algorithms for both non-coordinated and coordinated policies as follows:

Algorithm 1 (Non-coordinated policy from buyer's perspective):
Step 1: Set $BP_{\max} = 0$ and $n = 1$.
Step 2: Compute $\tilde{Q}_b^*$ [from (17.a) & (20.a)].
Step 3: Compute $\widehat{BP}_{nc}^*$ [from (17.b)&(20.b)], $\widetilde{VP}_{nc}^*$ [from (17.c)&(20.c)] and $\widetilde{AP}_{nc}^*$ [from (17.d)&(20.d)].
Step 4: If $BP_{\max} < \widehat{BP}_{nc}^*$, Set $BP_{\max} = \widehat{BP}_{nc}^*$ and Set $n = n + 1$ and go to Step 2. Else, go to Step 5.
Step 5: Stop

Algorithm 2 (Non-coordinated policy from vendor's perspective):
Step 1: Set $VP_{\max} = 0$ and $n = 1$.
Step 2: Compute $\widetilde{QP}_v^*$ [from (18.a) & (21.a)].
Step 3: Compute $\widetilde{VP}_{nc}^*$ [from (18.b)&(21.b)], $\tilde{B}_{nc}^*$ [from (18.c)&(21.c)] and $\widetilde{AP}_{nc}^*$ [from (18.d)&(21.d)]
Step 4: If $VP_{\max} < \widetilde{VP}_{nc}^*$, Set $VP_{\max} = \widetilde{VP}_{nc}^*$ and Set $n = n + 1$ and go to Step 2. Else, go to Step 5.
Step 5: Stop.

Algorithm 3 (Coordinated policy):
Step 1: Set $AP_{\max} = 0$ and $n = 1$.
Step 2: Compute $\tilde{Q}^*$ [from (19.a) & (22.a)].
Step 3: Compute $\tilde{C}_5^*$ [from (19.b) & (22.b)].
Step 4: Compute $\widehat{BP}_c^*$ [from (19.c)&(22.c)], $\widetilde{VP}_c^*$ [from (19.d)&(22.d)] and $\widetilde{AP}_c^*$ [from (19.e)&(22.e)].
Step 5: If $AP_{\max} < \widetilde{AP}_c^*$, Set $AP_{\max} = \widetilde{AP}_c^*$ and set $n = n + 1$ and go to Step 2. Else go to Step 6.
Step 6: Stop

NUMERICAL EXAMPLES:

Model with fuzzy demand:

To solve the problem, we assume the crisp parameter-values as follows:

$C_0 = 500, C_1 = 100, C_2 = 25, C_3 = 20, C_4 = 25, C_6 = 30, h_b = 5, h_v = 4, \frac{1}{p} = 3200$ and $\lambda = 1.0$.

And the TFN for the annual demand is $\tilde{D} = (800, 1000, 1200)$.

Using centroid method , the solution for the model is obtained for different values of n, as given in Table 1.

Table 1: Optimal results for different values of n

n^*	Q^*	AC_c^*
1	445.28	7210.98
2	267.83	7589.49
3	195.20	7710.03
4	155.07	7757.65
5	129.47	7774.53
6	111.68	7775.69
7	98.58	7767.83

Now, we compare the results of coordinated policy with those of non-coordinated policy in Table 2a and Table 2b.

Table 2a: Optimal results of non-coordinated (buyer's perspective) and coordinated policies

Non-coordinated policy				Coordinated policy				
Q_b^*	BP_{nc}^*	VP_{nc}^*	AC_{nc}^*	Q^*	C_5	BP_c	VP_c	AP_c^*
128.66	4355.68	3396.04	7751.72	111.68	25.01	4356.17	3419.52	7775.69

Table 2b: Optimal results of non-coordinated (vendor's perspective) and coordinated policies

Non-coordinated policy				Coordinated policy				
Q_v^*	VP_{nc}^*	BP_{nc}^*	AP_{nc}^*	Q^*	C_5	BP_c	VP_c	AP_c^*
183.90	3417.41	4349.16	7766.57	111.68	25.01	4356.17	3419.52	7775.69

Observation: From Tables 2a and 2b, we see that the total average profit obtained in coordinated policy is greater than those were obtained in non-coordinated policy (7775.69 vs. 7751.72 and 7775.69 vs. 7666.57). So, there is a profit gain of 23.97 units (against non-coordinated policy with buyer's perspective) and 9.12 units (against non-coordinated policy from vendor's perspective) if we apply coordinated approach. Also, the individual profits increase in coordinated policy.

Model with fuzzy costs:

In this case, the crisp parameter-values are as follows:

$D = 1000, \frac{1}{p} = 3200$ and $\lambda = 1.0$ and the TFNs for the possible costs are:

$\tilde{C}_o = (450, 500, 550)$, $\tilde{C}_1 = (90, 100, 110)$, $\tilde{C}_2 = (22.5, 25, 27.5)$, $\tilde{C}_3 = (18, 20, 22)$, $\tilde{C}_4 = (22.5, 25, 27.5)$, $\tilde{C}_6 = (27, 30, 33)$, $\tilde{h}_b = (4.5, 5, 5.5)$ and $\tilde{h}_v = (3.6, 4, 4.4)$.

The optimal results for different values of n are shown in Table 3.

Table 3: Optimal results for different values of n

n	Q^*	AP_c^*
1	447.21	7204.92
2	268.74	7581.32
3	195.70	7700.54
4	155.36	7747.22
5	129.66	7763.37
6	111.80	7763.93
7	98.66	7755.56

From Table 3, we see that the optimal number of shipments (n) is 6 and the corresponding profit of the coordinated policy is 7763.93 units. We now compare the results of coordinated policy with those of non-coordinated policy in Tables 3a and 3b.

Table 3a: Optimal results of non-coordinated (buyer's perspective) and coordinated policies

Non-coordinated policy				Coordinated policy				
Q_b^*	BP_{nc}^*	VP_{nc}^*	AP_{nc}^*	Q^*	C_5	BP_c	VP_c	AP_c^*
129.10	4354.50	3386.26	7740.76	111.80	25.00	4366.19	3397.84	7763.93

Table 3b: Optimal results of non-coordinated (vendor's perspective) and coordinated policies

Non-coordinated Approach				Coordinated Approach				
Q_v^*	VP_{nc}^*	BP_{nc}^*	AP_{nc}^*	Q^*	C_5	BP_c	VP_c	AP_c^*
162.22	3458.90	4286.23	7745.12	111.80	25.00	4366.09	3397.84	7763.93

Observation: Tables 3a and 3b show that the average profit obtained in coordinated policy is greater than those were obtained in non-coordinated policy (7763.93 vs. 7740.76 and 7763.93 vs. 7745.12). So, there is a profit gain of 23.172 units (against non-coordinated policy from buyer's perspective) and 18.81 units (against non-coordinated policy from vendor's perspective) if we adopt coordinated approach. The individual profits also increase in coordinated policy.

Table 4: Effects of changes in the parameter-values on the optimal results

Parameters	% change in parameter-values	% change in			
		n	Q	C_5	AP_c
D	+50	+16.67	+16.34	-0.002	+60.32
	+20	0.0	+12.39	+0.029	+23.81
	-20	-16.67	+1.54	+0.061	-23.28
	-50	-33.33	-5.72	+0.058	-56.98
$\dfrac{1}{p}$	+50	-16.67	+11.98	+0.056	-1.03
	+20	-16.67	+13.93	+0.063	-0.52
	-20	0.0	+3.28	+0.030	+0.91
	-50	+66.67	-19.53	-0.159	+4.46
λ	+50	0.0	0.0	-0.009	0.0
	+20	0.0	0.0	-0.004	0.0
	-20	0.0	0.0	+0.005	0.0
	-50	0.0	0.0	+0.015	0.0

We now examine the sensitivity of the optimal results obtained in the coordinated approach with fuzzy costs to changes in the parameters D, p and λ. While computing, the value of a single parameter is changed from -50% to $+50\%$ and the other parameters are kept unchanged. The results are shown in Table 4. On the basis of the results shown in Table 4, the following observations can be made.

- The number of replenishments n is slightly sensitive to changes in D and highly sensitive to the changes in $1/p$. It is invariant on the changes in λ.
- C_5 is almost insensitive to all the parameters.
- Q is slightly sensitive to D and $1/p$, but remains constant to the changes in λ.
- T_c increases highly with the increment of D and decreases slightly with the increment of $1/p$.

CONCLUSION:

Incorporating fuzzy set theory into supply chain models enables us to handle the vagueness of the parameters in real world applications. In this article, we first develop the model considering the annual demand as fuzzy and the other parameters as crisp. Next, we consider all the cost parameters as fuzzy and other parameters as crisp. Here, we have used symmetric TFNs to represent the imprecise annual demand, set up cost, ordering cost,

transportation cost, per unit production cost, un-discounted per unit sell price of the vendor, per unit sell price of the buyer, holding cost per unit per unit time for the vendor, holding cost per unit per unit time for the buyer and derived the average profit of the system in fuzzy sense. We then applied centroid method for defuzzification of the fuzzy profit. It is observed from the numerical study that the joint profit for the coordinated policy is higher than those are obtained in the non-coordinated policy. This emphasizes that coordination between vendor and buyer is very much essential to obtain better optimal results. The model may be extended by considering the other parameters as fuzzy and different defuzzification methods may also be used to obtain the average profit in crisp. Another extension of this work may be possible by allowing shortages in buyer's inventory or considering the production system imperfect.

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