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# Replenishment Policy for Price Sensitive Demand

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**ABSTRACT**

*One can manipulate the price of items to generate the excess demand. This paper developed a dynamic pricing policy for growing market. The replenishment policy also has been suggested to inventory managers. It is shown that dynamic pricing policy outperform to static pricing policy for price sensitive demand in growing market. A numerical example is given to illustrate the proposed model.*

**Keywords:** Optimal number of price settings, price sensitive demand, inventory, deterioration, holding cost.

## INTRODUCTION:

Marketing and pricing is an effective tool to establish the product in the market. When to reduce the price and when increase is most debatable task, but in deckling market reduce in price is always promoted by inventory managers. Many products lose their utility and quality after duration, like fashion products have limited sale season; they are out-dated or out of fashion thereafter. Many food products may be rotten after time duration therefore a pricing policy is required to sellout the entire stock before the start of next sale season. It needs to implement special sale or price reduction or both for a short duration. It is being advised to managers to control and maintain the inventory according to future potential of demand influenced by policies of inventory management and retailers. Nigwal and Khedlekar (2018) developed coordination policy for reverse supply chain which consists collector, retailer and re-manufacture assuming supply of used product depends on acquisition price and demand of used product depends on retail price. Expenditure sources like ordering cost, safety features, lead time and numbers of lots are the integral parts of decision making. Lo (2007) presents inventory model focusing on ordering cost, safety feature, lead time and number of lots for deteriorating item.
Dividing demand rate into multiple segments, Shukla and Khedlekar (2010) introduce three-component demand rate based on marketing policies for the newly launched deteriorating product. Widyadana and Wee (2010) solved the problem by genetic algorithm method for unreliable production system with price dependent demand and suggest pricing policy to minimize the loss in profit due to machine unavailability. Ziaee et al. (2011) showed that if vendor managed problem analytically solved then solution may outperform to the traditional solution. Alfares et al. (2005) develop product-inventory model for both deteriorating item and deteriorating process considering some realistic aspects in their model such as varying demand, production rates, quality, and malignance cost. Some important contributions on inventory modeling are Mohammad et al. (2016), Kannadasan et al. (2017), Ghosh (2017).
Roy and Chaudhary (2007) introduce the concept of sale promotion in festival for the clearance of over due stock and compared it without special sale and found this outperforming to earlier. The back order and partial lost sale is investigated due to Lo et al. (2008) with the impact of leading time on optimal policy and safety. Federgruen and Heching (1999) design the replenishment strategy when price of item faces uncertainty and follows a general stochastic demand function. They analyzed the simultaneous price and the inventory in an incapacitated system with the use of stochastic demand for single item. The aspect of inflation and delay in payment to vendors has been attempted by many authors. A research overview presented by Elmaghraby and

Keskinocak (2003) is based on the above problem incorporating the future planning as it jointly determines the dynamic pricing and order level both.

Bernstein and Federgruen (2003) analyze the constant demand-rate and solved the problem of VMI (Vendor Management Inventory) where the replenishment decision-making is transferred to the supplier, but the retailer is able to make own pricing decisions. We refer literature on pricing policy by Shukla at el. (2012, 1013, 2016), Maragatham and Gnanvel (2017), Khedlekar (2012), Maheswari and Elango (2017) You (2007). Sinha (2008) estimates parameters in regression model and the problem of ranking of business schools solved by Karush-Kuhn-Tucker optimization method. Results are found satisfactory in quality.

You (2005) discuss an inventory policy for the product with price and time dependent demand. He obtained the order size and optimal price both when decision maker has an opportunity to adjust price before the end of sale season. In this paper we consider the constant demand and price sensitive demand and present a dynamic pricing policy.

## MATHEMATICAL MODEL:

Let us assume that one can purchase the items at rate \$c per unit and sells it at rate $P(p_1, p_2 \dots p_n)$ per unit. Suppose time cycle is L, divided into n equal parts as T = L/n and prices ine subsequent intervals are $p_1, p_2, \dots\dots p_n$. His aim is to find the prices in different interval in growing market and earn more. Suppose the quantity q of product purchased at rate \$c per unit and holding cost is h unit per unit time. Product deteriorates at rate $\theta$, the total revenues, deteriorated units and profit in the business is R, D and F respectively. We design the proposed model under assumption that shortages are not allowed, replenishment rate is infinite and lead time is zero. Following notations are used to develop the model. Assumed that demand of product is $d(t, p_i) = (a - \beta\, p_i)$

where i =1, 2….n, α >0, and $\beta > 0$ are fix for a given business setup such that $\alpha - \beta p_i > 0$ and t >0.

$$s_i = \sum_{j=1}^{i} \int_{(j-1)T}^{jT} d(t, p_j)\, dt = \alpha i T - \beta T \sum_{j=1}^{i} p_j \tag{1}$$

The total sold amount over time horizon (L= nT) is

$$q = s_n = \alpha n T - \beta T \sum_{j=1}^{n} p_j \tag{2}$$

The objective is to find out optimal number of change in selling price, respective selling prices alongwith optimum profit. Suppose $I_i(t)$ is on hand inventory at time t. The demand function satisfies following differential equation in time interval [(i-1)T, iT].

$$\frac{d}{dt} I_i(t) = -d\left((i-1)T + t, p_j\right) = -\left(\alpha - \beta\, p_j\right) \tag{3}$$

with boundary condition $I_i(0) = q - s_{i-1}$ and put

$$I_i(t) = q - \alpha - T\alpha(i-1) + \beta T \sum_{j=1}^{i-1} p_j + \beta t p_j \tag{4}$$

The holding $H$ is:

$$H = h \sum_{i=1}^{n} \int_0^T I_i(t) dt \tag{5}$$

$$= h\left\{ T^2 \alpha \sum_{i=1}^{n} (n - i + 1) - \alpha T + \frac{T^2}{2} \beta \sum_{i=1}^{n} p_j \right\}$$

Total deteriorated units $D(n, p^n)$ over time [0, T] are

$$D = \theta \sum_{i=1}^{n} \int_0^T I_i(t) dt \quad = \theta \left\{ T^2 \alpha \sum_{i=1}^{n} (n - i + 1) - \alpha T + \frac{T^2}{2} \beta \sum_{i=1}^{n} p_j \right\} \tag{6}$$

Due to deterioration required quantity in system will be

$$q_1 = q + D(n, p^n) = s_n + D(n, p^n)$$

Total earned revenue $R(n,\mathrm{p}^{n})$ over time L is;

$$R(n,\mathrm{p}^{n})=\sum_{j=1}^{n}p_j\int_{(j-1)T}^{jT}d(t,p_j)\,dt$$

$$=\alpha\sum_{i=1}^{n}p_j-\beta T\sum_{i=1}^{n}p_j^2 \tag{7}$$

Net profit $F(n,\mathrm{p}^{n})=R-H-cq'-nc_0$

$$=\alpha\sum_{i=1}^{n}p_i-\beta T\sum_{i=1}^{n}p_i^2-(h+\theta c)\left\{\alpha T^2(n-i+1)\alpha T+T^2\beta p_i\right\}$$

$$=Tc\left(\alpha n-\beta\sum_{i=1}^{n}p_i\right)-nc_0 \tag{8}$$

Our objective is to maximize $F(n,\mathrm{p}^{n})$ under the condition that

$p_i<\frac{a}{\beta}$ for demand function $a-\beta\, p_i>0$

$$\text{or }\; p_j-\frac{a}{\beta}+Z_j^{\,2}=0 \;\text{ where }\; Z_j^{\,2}=A_i \tag{9}$$

The suggested objective function is $L(n,\mathrm{p}^{n})$ under the condition (9) with using Lagrangian multiplier $\lambda_j$ could be expressed as

$$L(n,\mathrm{p}^{n})=F(n,\mathrm{p}^{n})-\sum_{j=1}^{n}\lambda_j\left(p_j-\frac{a}{\beta}+Z_j^{\,2}\right),\; n\le N_{\max} \tag{10}$$

If $A_i>0$, optimal price for interval $\left[(i\text{-}1)T\;,iT\right]$ is

$$\frac{\partial}{\partial p_i}L(n,\mathrm{p}^{n})=0 \tag{11}$$

$$p_{i_1}=\frac{T}{2}(h+A\theta)\left(i-\frac{1}{2}\right)+\frac{A}{2}+\frac{\alpha}{2\beta}-\frac{\lambda_i}{2\beta T} \tag{12}$$

If $A_i\le 0$, then $\frac{\partial}{\partial\lambda_i}L(n,\mathrm{p}^{n})=-\left(p_i-\frac{a}{\beta}+Z_i^{\,2}\right)$

$$p_{i2}=\frac{a}{\beta}$$

if $A_i>0$, then calculate $p_{i_1}$, from (12),

if $A_i\le 0$, then calculate $p_{i_2}$, from (14), then calculate respective $R(n,p^n)$, $F(n,p^n)$, D for each n and i.

obtain n* for that $F(n,p^n)$ is optimal, declare n* ( number of price setting).

**An Example:**

We have considered an example to illustrate the model, If α = 200, β=0.2 θ = 0, $c_0$=50, A=50 and time horizon L=90. See solution in table 1.

**Table 1: Optimal price settings**

| n | $H(n,p^n)$ | $R(n,p^n)$ | $K(n,p^n)$ | $F(n,p^n)$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2002.02 | 4485507.47 | 426514.52 | 4058992.95 | 528.38 | | | | | | |
| 2 | 1596.33 | 4485456.21 | 426158.83 | 4059297.38 | 526.69 | 530.06 | | | | | |
| 3 | 1461.21 | 4485446.72 | 426073.71 | 4059373.01 | 526.13 | 528.38 | 530.63 | | | | |
| 4 | 1393.66 | 4485443.40 | 426056.16 | 4059387.23 | 525.84 | 527.53 | 529.22 | 530.91 | | | |
| 5 | 1353.14 | 4485441.86 | 426065.64 | 4059376.22 | 525.68 | 527.03 | 528.38 | 529.73 | 531.08 | | |
| 6 | **1326.13** | **4485441.02** | **426088.63** | **4059352.39** | **525.56** | **526.69** | **527.81** | **528.94** | **530.06** | **531.19** | |
| 7 | 1306.84 | 4485440.52 | 426119.34 | 4059321.18 | 525.42 | 526.45 | 527.41 | 528.38 | 529.34 | 530.30 | 531.27 |

In above example optimal price setting is n*=6, since profit $F(n,p^n)$ is greater than for n = 1, 2, 3, 4, 5 and 7 and respectively prices in different intervals are $p_1$=525.56, $p_2$= 526.69, $p_3$=527.81, $p_4$=528.94, $p_5$=530.06, $p_6$=531.19.

## CONCLUSION:

A dynamic pricing policy is developed for growing market of price sensitive item. The proposed pricing policy is better than a constant pricing policy. Customers may responses better even selling price increase in growing market. Inventory managers are suggested to decide their most frequent cost and deterioration level to get maximum profit in view of suggested robust ranges of the model parameters. It is advised to inventory managers to keep less deterioration rate, rather than lowering the other cost for earning more profit. Also, the high initial demand and low holding cost together provide higher profit to the inventory holder. One can incorporate ameliorating in place of deterioration and dynamic replenishment in present scenario in deckling market.

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